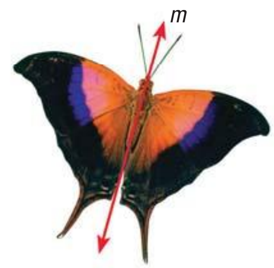
9.6 Identify Symmetry

Before You reflected or rotated figures.
Now You will identify line and rotational symmetries of a figure.
Why? So you can identify the symmetry in a bowl, as in Ex. 11.

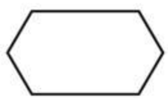
Key Vocabulary
- **line symmetry**
- **line of symmetry**
- **rotational symmetry**
- **center of symmetry**

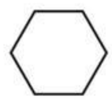
A figure in the plane has **line symmetry** if the figure can be mapped onto itself by a reflection in a line. This line of reflection is a **line of symmetry**, such as line *m* at the right. A figure can have more than one line of symmetry.

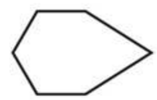


EXAMPLE 1 Identify lines of symmetry

How many lines of symmetry does the hexagon have?

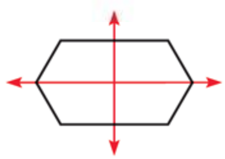
a.


b.


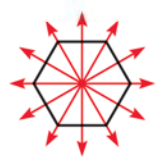
c.


Solution

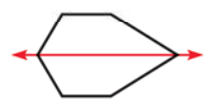
a. Two lines of symmetry



b. Six lines of symmetry



c. One line of symmetry



REVIEW REFLECTION
Notice that the lines of symmetry are also lines of reflection.

Animated Geometry at classzone.com

✓ GUIDED PRACTICE for Example 1

How many lines of symmetry does the object appear to have?

1.


2.

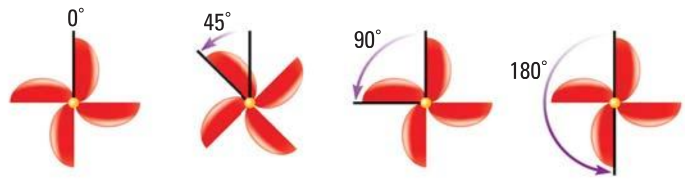

3.


4. Draw a hexagon with no lines of symmetry.

ROTATIONAL SYMMETRY A figure in a plane has **rotational symmetry** if the figure can be mapped onto itself by a rotation of 180° or less about the center of the figure. This point is the **center of symmetry**. Note that the rotation can be either clockwise or counterclockwise.

REVIEW ROTATION
For a figure with rotational symmetry, the *angle of rotation* is the smallest angle that maps the figure onto itself.

For example, the figure below has rotational symmetry, because a rotation of either 90° or 180° maps the figure onto itself (although a rotation of 45° does not).

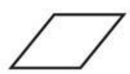


The figure above also has *point symmetry*, which is 180° rotational symmetry.

EXAMPLE 2 Identify rotational symmetry

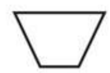
Does the figure have rotational symmetry? If so, describe any rotations that map the figure onto itself.

a. Parallelogram



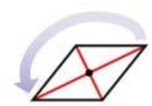
b. Regular octagon

c. Trapezoid

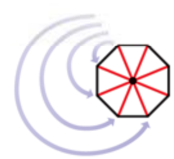


Solution

a. The parallelogram has rotational symmetry. The center is the intersection of the diagonals. A 180° rotation about the center maps the parallelogram onto itself.



b. The regular octagon has rotational symmetry. The center is the intersection of the diagonals. Rotations of 45°, 90°, 135°, or 180° about the center all map the octagon onto itself.

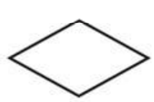


c. The trapezoid does not have rotational symmetry because no rotation of 180° or less maps the trapezoid onto itself.

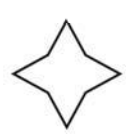
✓ GUIDED PRACTICE for Example 2

Does the figure have rotational symmetry? If so, *describe* any rotations that map the figure onto itself.

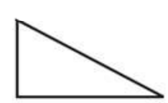
5. Rhombus



6. Octagon

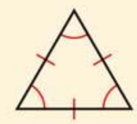


7. Right triangle



★ EXAMPLE 3 Standardized Test Practice

Identify the line symmetry and rotational symmetry of the equilateral triangle at the right.



Ⓐ 3 lines of symmetry, 60° rotational symmetry

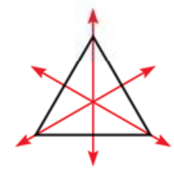
Ⓑ 3 lines of symmetry, 120° rotational symmetry

Ⓒ 1 line of symmetry, 180° rotational symmetry

Ⓓ 1 line of symmetry, no rotational symmetry

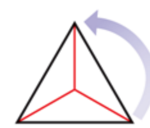
Solution

ELIMINATE CHOICES
An equilateral triangle can be mapped onto itself by reflecting over any of three different lines. So, you can eliminate choices C and D.



The triangle has line symmetry. Three lines of symmetry can be drawn for the figure.

For a figure with s lines of symmetry, the smallest rotation that maps the figure onto itself has the measure $\frac{360°}{s}$. So, the equilateral triangle has $\frac{360°}{3}$, or 120° rotational symmetry.



▸ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Example 3

8. *Describe* the lines of symmetry and rotational symmetry of a non-equilateral isosceles triangle.

9.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 13, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 13, 14, 21, and 23

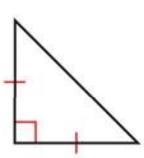
SKILL PRACTICE

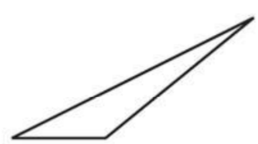
1. **VOCABULARY** What is a *center of symmetry*?

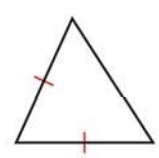
2. ★ **WRITING** Draw a figure that has one line of symmetry and does not have rotational symmetry. Can a figure have two lines of symmetry and no rotational symmetry?

EXAMPLE 1 on p. 619 for Exs. 3–5

LINE SYMMETRY **How many lines of symmetry does the triangle have?**

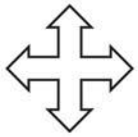
3. 

4. 

5. 

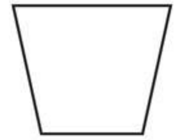
EXAMPLE 2 on p. 620 for Exs. 6–9

ROTATIONAL SYMMETRY **Does the figure have rotational symmetry? If so, *describe* any rotations that map the figure onto itself.**

6. 

7. 

8. 

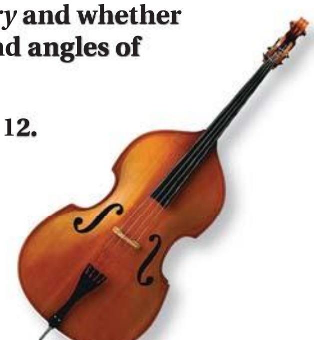
9. 

EXAMPLE 3 on p. 621 for Exs. 10–16

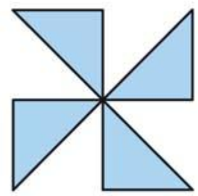
SYMMETRY **Determine whether the figure has *line symmetry* and whether it has *rotational symmetry*. Identify all lines of symmetry and angles of rotation that map the figure onto itself.**

10. 

11. 

12. 

13. ★ **MULTIPLE CHOICE** Identify the line symmetry and rotational symmetry of the figure at the right.



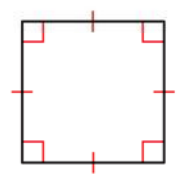
Ⓐ 1 line of symmetry, no rotational symmetry

Ⓑ 1 line of symmetry, 180° rotational symmetry

Ⓒ No lines of symmetry, 90° rotational symmetry

Ⓓ No lines of symmetry, 180° rotational symmetry

14. ★ **MULTIPLE CHOICE** Which statement best describes the rotational symmetry of a square?



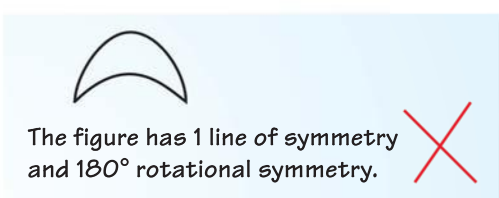
Ⓐ The square has no rotational symmetry.

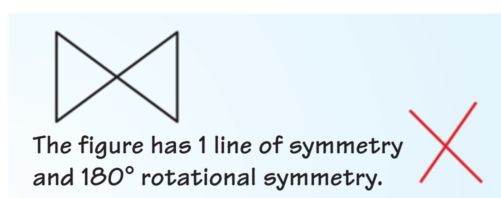
Ⓑ The square has 90° rotational symmetry.

Ⓒ The square has point symmetry.

Ⓓ Both B and C are correct.

ERROR ANALYSIS ***Describe* and correct the error made in describing the symmetry of the figure.**

15. 

16. 

DRAWING FIGURES **In Exercises 17–20, use the description to draw a figure. If not possible, write *not possible*.**

17. A quadrilateral with no line of symmetry

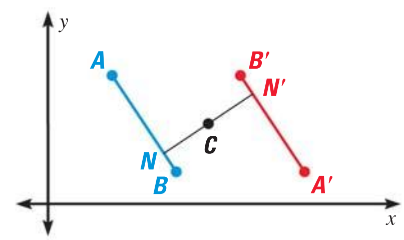
18. An octagon with exactly two lines of symmetry

19. A hexagon with no point symmetry

20. A trapezoid with rotational symmetry

21. ★ **OPEN-ENDED MATH** Draw a polygon with 180° rotational symmetry and with exactly two lines of symmetry.

22. **POINT SYMMETRY** In the graph, $\overline{AB}$ is reflected in the point C to produce the image $\overline{A'B'}$. To make a reflection in a point C for each point N on the preimage, locate N' so that $N'C = NC$ and N' is on $\overleftrightarrow{NC}$. *Explain* what kind of rotation would produce the same image. What kind of symmetry does quadrilateral $AB'A'B$ have?



23. ★ **SHORT RESPONSE** A figure has more than one line of symmetry. Can two of the lines of symmetry be parallel? *Explain*.

24. **REASONING** How many lines of symmetry does a circle have? How many angles of rotational symmetry does a circle have? *Explain*.

25. **VISUAL REASONING** How many planes of symmetry does a cube have?

26. **CHALLENGE** What can you say about the rotational symmetry of a regular polygon with n sides? *Explain*.

PROBLEM SOLVING

EXAMPLES 1 and 2
on pp. 619–620 for Exs. 27–30

WORDS Identify the line symmetry and rotational symmetry (if any) of each word.

27. MOW 28. RADAR 29. OHIO 30. pod

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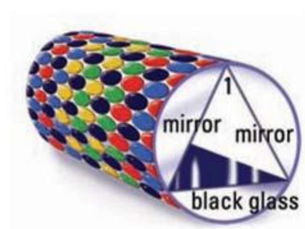
KALEIDOSCOPES In Exercises 31–33, use the following information about kaleidoscopes.

Inside a kaleidoscope, two mirrors are placed next to each other to form a V, as shown at the right. The angle between the mirrors determines the number of lines of symmetry in the image. Use the formula $n(m\angle 1) = 180°$ to find the measure of $\angle 1$ between the mirrors or the number n of lines of symmetry in the image.



Calculate the angle at which the mirrors must be placed for the image of a kaleidoscope to make the design shown.

31.

32.

33.

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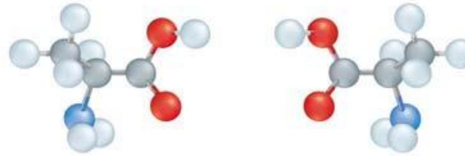
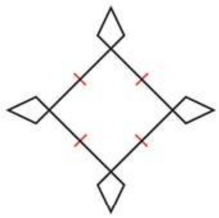
34. CHEMISTRY The diagram at the right shows two forms of the amino acid *alanine*. One form is laevo-alanine and the other is dextro-alanine. How are the structures of these two molecules related? *Explain*.



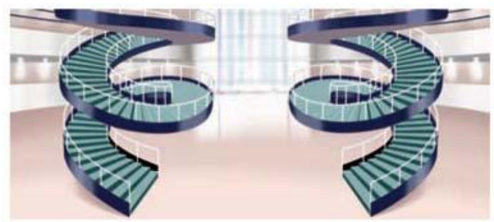
35. MULTI-STEP PROBLEM The *Castillo de San Marcos* in St. Augustine, Florida, has the shape shown.



a. What kind(s) of symmetry does the shape of the building show?

b. Imagine the building on a three-dimensional coordinate system. Copy and complete the following statement: The lines of symmetry in part (a) are now described as __?__ of symmetry and the rotational symmetry about the center is now described as rotational symmetry about the __?__.

36. CHALLENGE Spirals have a type of symmetry called spiral, or helical, symmetry. *Describe* the two transformations involved in a spiral staircase. Then *explain* the difference in transformations between the two staircases at the right.



MIXED REVIEW

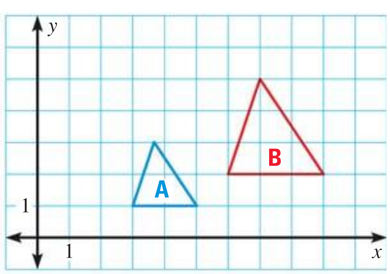
PREVIEW
Prepare for Lesson 9.7 in Exs. 37–39.

Solve the proportion. *(p. 356)*

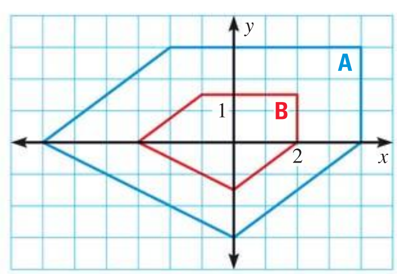
37. $\frac{5}{x} = \frac{15}{27}$ **38.** $\frac{a+4}{7} = \frac{49}{56}$ **39.** $\frac{5}{2b-3} = \frac{1}{3b+1}$

Determine whether the dilation from Figure A to Figure B is a *reduction* or an *enlargement*. Then find its scale factor. *(p. 409)*

40.



41.



Write a matrix to represent the given polygon. *(p. 580)*

42. Triangle A in Exercise 40
43. Triangle B in Exercise 40
44. Pentagon A in Exercise 41
45. Pentagon B in Exercise 41